Efficient TLM Diakoptics for Separable Structures

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***Abstract*—An efficient yet rigorous application of diakoptics to TLM simulation of discontinuities in homogeneously filled waveguides is proposed. The method, based on the expansion of the time-domain Green's function into frequency independent eigenfunctions, leads to a dramatic reduction of the numerical effort when compared to the standard Johns Matrix approach. Numerical results show that this new approach provides wide band absorbing boundaries for waveguide problems where several modes (whether propagating or evanescent) are present. In this way, the computational domain is also reduced to just a small region around the discontinuity, with the absorbing boundaries placed just a few cells away.**

I. Introduction

IT IS NOW commonly acknowledged that discontinuities of complex shape in waveguide can be effectively analyzed by time domain methods such as TLM. In fact, their inherent discretization allows the modeling of discontinuities of arbitrary geometry almost effortlessly.

However, to obtain accurate results, particular care must also be used when choosing appropriate absorbing boundary conditions (ABC's) to terminate the computational domain. In the case of waveguide components the side walls are generally represented by perfect electric conductors. Thus only the input and output planes need to be modeled explicitly. For example absorbing boundary conditions such as those reported in [1] could be implemented on these planes. These ABC's work well for a given incidence angle of the field and therefore, in a waveguide, for a given frequency. In practical time domain analysis the excitation of the electromagnetic fields inside a waveguide covers generally a certain bandwidth. Hence, local approximate boundary conditions cannot be employed to compute S-parameters over a wide frequency band in a single run.

As an alternative, a rigorous approach based on the use of diakoptics [2] can be applied. In time domain diakoptics, waveguide discontinuities are analyzed by partitioning the circuit into sub-domains that can be analyzed independently and then connected together. The modeling of ABC's is thus accomplished by pre-simulating a semi-infinite waveguide and by using the time domain response of such a structure to terminate the computational domain. This procedure is extremely advantageous since it allows pre-processing of standard subsections and thus to limit numerical analysis to the specific part of interest, for example the region immediately around a discontinuity. However, when the boundaries of the sub-domain are close to the discontinuity, the full Johns matrix [3] characterizing the dispersive interface impedance in the time domain must be evaluated and then convolved with the stream of incident impulses. This procedure has the following drawbacks:

1) long computation time is needed to calculate the various convolutions at the nodes on the boundaries;
2) large storage is also required to store the 3-D Johns matrix.

To overcome these drawbacks, we exploit some properties of the structures under investigation. In particular, for waveguides with frequency independent modal field distribution, the numerical effort can be reduced dramatically by representing the TLM incident fields simply as a superposition of modes which are individually matched in the time domain.

It should be mentioned that this procedure has been already introduced for a single mode by Eswarappa, Costache, and Hoefer [4]. This technique works very well when just one mode is propagating. However, the absorbing walls must be placed far enough from the discontinuity so that only the dominant mode prevails. The present approach is the natural extension of this procedure for higher order modes, allowing us to place the absorbing walls close to the discontinuity where these modes are still significant [5].

II. Theory

In order to explain the theory let us refer to a homogeneously filled waveguide as depicted in Fig. 1. It is well known that the *frequency domain Green's function* of a waveguide with frequency independent modes $\Phi_p(r)$ is of the type

$$\tilde{g}(r, r', \omega) = \sum_{p=1}^{\infty} \tilde{\chi}_p(\omega)\Phi_p(r)\Phi_p(r') \qquad (1)$$

where r, r' are points on the same cross-section S, ω is the angular frequency, and $\tilde{\chi}_p(\omega)$ depends on the particular excitation/observation field quantities used. As an example, when we excite the field by means of a current source and

Manuscript received April 5, 1993; revised July 13, 1994. This work was supported by the Natural Sciences and Engineering Research Council of Canada, the Science Council of British Columbia, MPR Teltech Inc., Burnaby, BC, and the University of Victoria.

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IEEE Log Number 9408569.

0018–9480/95$04.00

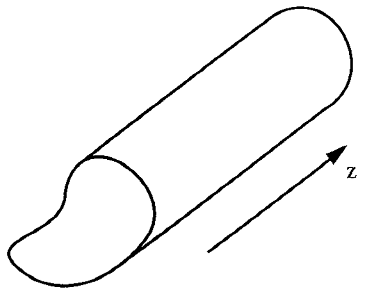

Fig. 1. Geometry of a homogeneously filled waveguide.

observe the electric field, $\tilde{\chi}_p(\omega)$ is of impedance type. In particular, if we consider a parallel plate waveguide with just TE_{n0} modes present then $\tilde{\chi}_p(\omega)$ is the well known expression $\tilde{\chi}_p(\omega) = \frac{j\omega\mu_0}{\beta_p}$ given in [10].

When the modes are frequency independent, the *time domain Green's function* is easily obtained from (1) as

$$g(r, r', t - t') = \sum_{p=1}^{\infty} \chi_p(t - t')\Phi_p(r)\Phi_p(r'). \qquad (2)$$

Now $\chi_p(t)$ represents the response of the waveguide in the time domain when the pth mode is excited. Its nature still depends on the particular excitation/observation quantities. In TLM we are interested in the relation between the incident and the reflected waves, $V^i(r,t)$ and $V^r(r,t)$, respectively. Such waves are modeled as a superposition of impulses $\Pi_n(r)$ as shown in (3) and (4). The sum of such waves leads to the electromagnetic field in a given cross-section. The same field can also be seen as a superposition of modes (Fig. 2)

$$V^i(r,t) = \sum_{n=1}^{N} V^i(n,t)\Pi_n(r) \qquad (3)$$

$$V^r(r,t) = \sum_{n=1}^{N} V^r(n,t)\Pi_n(r). \qquad (4)$$

In the frequency domain the relation between incident and reflected waves is simply given by the reflection coefficient $\Gamma(\omega)$ in a certain waveguide cross section. In the time domain that relation becomes a convolution of the incident wave with the impulse response of the waveguide.

$$V^r(r,t) = \int_s g(r, r', t - t') * V^i(r', t')\, dr' \qquad (5)$$

where $*$ denotes numerical convolution.

The integration over the cross-section S is necessary in order to take into account interaction between different points, r and r', in the same cross-section.

By inserting (2) and (3) into (5), and by defining the coupling coefficient between modes and TLM impulse functions

$$C_{np} = \int_s \Pi_n(r')\Phi_p(r')\, dr'$$

we obtain:

$$V^r(r,t) = \sum_{p=1}^{P} \Phi_p(r)\chi_p(t - t') * \left[\sum_{n=1}^{N} C_{np}V^i(n,t')\right]. \qquad (6)$$

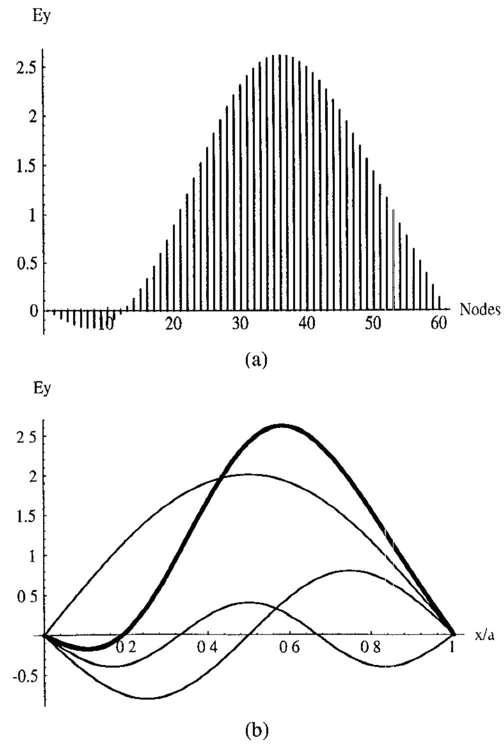

Fig. 2. (a) Field in the waveguide cross-section as a superposition of TLM impulses. (b) Field in the waveguide cross-section as a superposition of modes.

The spatial Fourier analysis performed by the matrix $\mathbf{C}$ allows us to separate the overall field into a sum of modes. In particular, the term $\sum_{n=1}^{N} C_{np}V^i(r,t')$ extracts from the incident wave $V^i(r,t)$ (superposition of all incident modes) the amplitude of the incident pth mode. The pth mode so isolated is convolved with its reflected wave time domain modal response $\chi_p(t')$, in order to obtain the amplitude of the reflected pth mode. The overall reflected wave is then recomposed by adding all the reflected modes. The recomposition is done by means of the same matrix $\mathbf{C}$ used now for a spatial Fourier synthesis.

When the incident wave is represented by just one mode, only one term of the sum over p in (6) is different from zero. Hence the reflected wave is also represented by just that mode. This corresponds to the fact that the impulse response of a mode is independent of the impulse response of other modes. However, this condition holds only for uniform wave guides where no coupling between modes takes place. The sum over the number of modes in (6) is truncated after P modes since even at a small distance from a discontinuity the field in the waveguide can be generally represented as a superposition of just the first few modes.

The reflected wave impulse response of the pth mode $\chi_p(t')$ is computed with the diakoptic procedure used in [8]. An impulse in time, with the spatial distribution of the pth mode, is injected into a semi-infinite empty waveguide. The reflected stream of impulses generated during such a pre-simulation is then stored. The process is repeated for all the modes accounted for in the absorbing boundary. Note that it is necessary to compute and store such Johns streams only once for a certain waveguide. They are then introduced

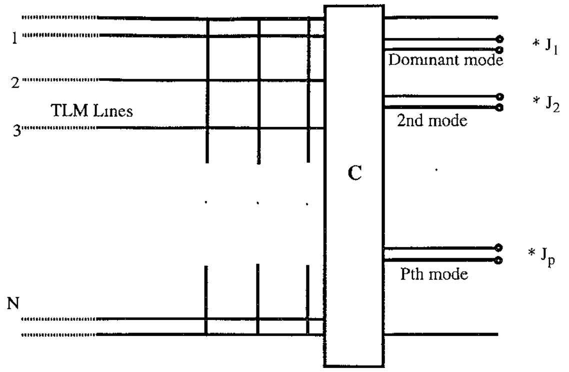

Fig. 3. Network representation of the transformation from the TLM transmission lines to transmission lines corresponding to modes in the waveguide.

in (6) to terminate the P modes present in the vicinity of a discontinuity. Also the matrix $\mathbf{C}$ is computed only once for a given waveguide after the modes have been isolated. For a rectangular waveguide the elements of this matrix are known in closed form, while for the general case a numerical integration is necessary.

Equation (6) is linked with the transmission line representation of our problem. Essentially the whole process described above has a simple interpretation in terms of the network in Fig. 3 where the ports on the left are connected to the TLM transmission lines. Therefore, N ports are present on the left. The N voltages and currents, or the amplitudes of the incident and scattered waves on the various ports, sample the electric and magnetic field in the waveguide cross section. The same field distribution can also be described in terms of P modes. With respect to Fig. 3 the voltages and currents on the P ports on the right provide the amplitudes of the modes. All the ports on the right are independent, and each of them is connected to a properly terminated transmission line which delivers the corresponding Johns stream.

Note that in this way not only infinitely long waveguides can be simulated (matched condition), but also other terminations such as metallic walls or lossy dielectrics placed at a certain distance along the z direction, or any arbitrary dispersive impedance.

It is now convenient to multiply both sides of (6) by $\Pi_m(r')$ and to integrate over the whole cross-section S in order to recover the reflected impulse at the mth node; we obtain

$$V^r(m,t) = \sum_{p=1}^{P} C_{mp}\chi_p(t-t') * \left[\sum_{n=1}^{N} C_{np}V^i(n,t')\right]. \quad (7)$$

For computational purposes we introduce the arrays

$$\underline{V}^i(t) = \begin{bmatrix} V^i(1,t) \\ V^i(2,t) \\ \vdots \\ V^i(N,t) \end{bmatrix} \quad \text{and} \quad \underline{V}^r(t) = \begin{bmatrix} V^r(1,t) \\ V^r(2,t) \\ \vdots \\ V^r(N,t) \end{bmatrix} \quad (8)$$

containing the spatial configuration of impulses, the matrix $\mathbf{C}$ composed of the elements C_{np}, and the diagonal matrix $\mathbf{J}$ composed of the elements $\chi_p(t-t')$ so that we can write the entire absorbing process in a compact form as

$$\underline{V}^r(k) = \mathbf{CJ}(k') * \mathbf{C}^T \underline{V}^i(k-k') \quad (9)$$

where the time has been discretized as $t = k\Delta t$.

In order to match the field incident upon the boundary, (9) must be applied at every iteration. The matrices $\mathbf{C}$ and $\mathbf{J}$ are precomputed as described above. The decomposition of the incident wave into incident modes is done for every iteration k. The amplitude of each incident mode is stored so that at the next iteration, $k+1$, only one more decomposition is necessary. After the Pth convolution (one for each mode) the complete reflected wave is recomposed from the reflected mode amplitudes and spatially sampled.

III. Application and Numerical Results

The application to a two-dimensional problem is now quite straightforward. If we assume a TE_{10}-mode propagating in the waveguide and we discretize the guide with a TLM mesh [9] it can be seen that the impulses traveling in the z-direction will propagate toward the absorbing boundaries. Such impulses sample the field components in the waveguide. The eigenfunctions for the E_y component are well known

$$\Phi_n(r) = \Phi_n(x) = \sin\frac{n\pi x}{a}. \quad (10)$$

In order to apply the modal Johns matrix we must then decompose the wave incident upon the boundary into a sum of eigenfunctions of the kind $\sin\frac{n\pi x}{a}$, and we must convolve each mode separately, as described above, with the appropriate pre-computed modal Johns matrix.

The procedure can also be applied to a three-dimensional problem. The first step is to separate the impulses as belonging to a TE or TM configuration. Then each TE (or TM) configuration is separated into modes by means of the proper eigenfunction.

It is clear that we must obtain a compromise between the size of the TLM mesh surrounding the discontinuity and the number of modes we consider. If we place the absorbing boundary exactly on the discontinuity we must consider a large number of modes in order to describe the complex field around it. Each mode must be convolved with its Johns stream. That leads to a fast TLM simulation that, unfortunately, is slowed down by the large number of convolutions. In this case we obtain a TLM version of mode-matching but we do not take full advantage of the TLM capability of modeling complex discontinuities. As we move the absorbing walls away from the discontinuities, all higher order modes are progressively attenuated until (depending on the excitation) only the fundamental mode or the first few modes are present. In this case we need a larger mesh (larger computational time and memory occupation) but only one convolution. The best compromise is reached by moving the absorbing boundaries a few cells away from the discontinuity so that the TLM mesh is still small and

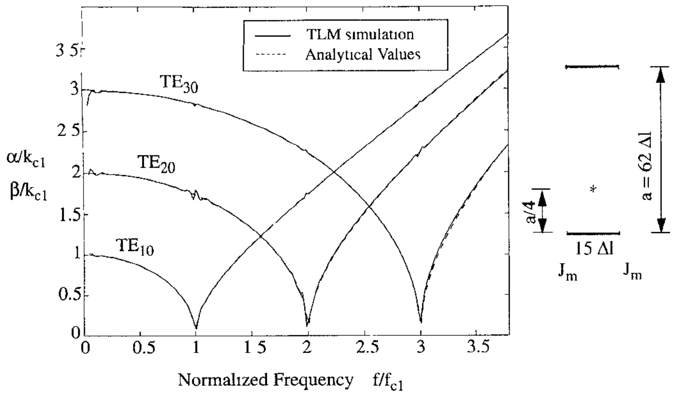

Fig. 4. Phase constant and attenuation for the TE_{10}, TE_{20} and TE_{30} in a WR (28) waveguide. Comparison between TLM and analytical values. J_m indicates the position of the modal ABC's; * indicates the position of the point source.

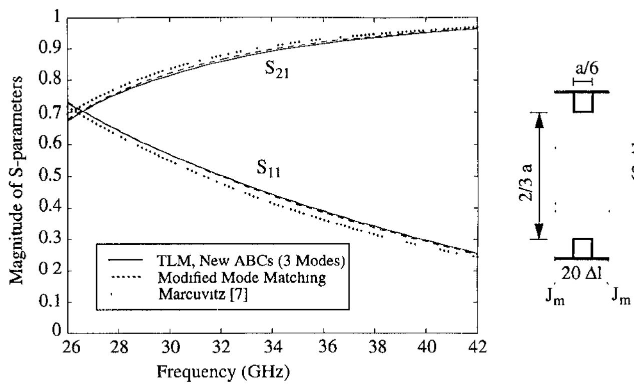

Fig. 5. Magnitude of S-parameters for a thick inductive iris.

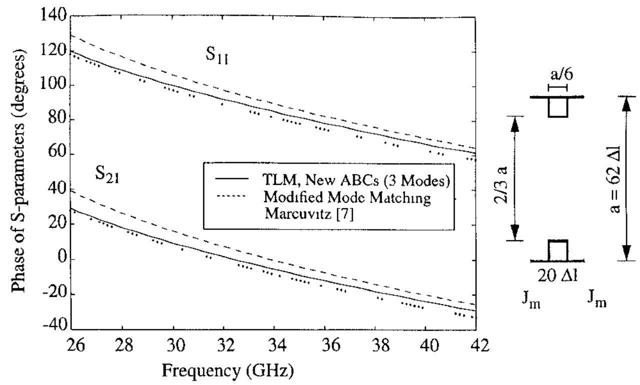

Fig. 6. Phase of S-parameters for a thick inductive iris.

the number of modes to be considered is reduced to 2 or 3. This compromise gives us a saving in computational time of one order of magnitude if compared to the classical (single mode) approach. In addition we have the following advantages:

—The modal absorbing boundaries are not sensitive to the excitation, so even if we excite a waveform with frequency content above the second mode cutoff (which would excite a propagating second mode) the results are not affected. This is particularly important when we want reliable results close to the second cutoff frequency. It leads to a very robust simulation where instabilities in the ABC's were never encountered.

—The signal travels very quickly through the small computational domain so that we can use relatively short Johns matrices.

In order to verify the capacity of the modal Johns matrix to match modes above and below cutoff, the propagation constant, $\beta(f)$, and the attenuation constant, $\alpha(f)$, of a WR (28) waveguide have been calculated. To compute the above parameters an empty waveguide has been discretized and excited by a point source in order to excite several modes, with a bandlimited excitation in time. The separation between modes allows us to obtain the phase constant and the attenuation for the first three modes with a single simulation. The results are shown in Fig. 4. In the frequency bands between the cutoff of the first modes they are within 0.5% of the analytical values, while the results close to the cutoff frequencies worsen (error of a few percent) due to the difficulty in modeling modes at the cutoff in the time domain.

The new modal Johns matrix has been used to determine the S-parameters of two discontinuities in a rectangular waveguide WR (28): a symmetric thick iris and an asymmetric rectangular post. The excitation is a signal with frequency content covering the dominant operating range of the waveguide. The choice of such a signal guaranties a faster convergence toward the solution. In addition, avoiding the frequencies at the cutoff eliminates the ringing of a mode with very low group velocity that remains in the mesh for a long time. This reduces the truncation error.

The thick iris has been studied with a mesh of size 20×62. Three modes have been considered in the absorbing process for absorbing planes placed $\lambda_0/20$ from the iris (aperture $a' = 2a/3$ and thickness $t = a/6$). The results have been compared with those obtained with a modified mode matching method and with those of Marcuvitz [7].

The results for magnitude and phase of the iris with aperture $a' = 2a/3$ and thickness $t = a/6$ are shown in Figs. 5 and 6. The good agreement in the phase of S-parameters shows that the reactive load of the iris due to the modes below cutoff has also been correctly simulated by the higher order modal Johns matrices.

The asymmetric transverse strip has been examined with the same discretization as the iris, matching all modes (even and odd) up to the TE_{50}. The number of modes considered in the ABC's has been determined by increasing the number of modes until a convergence in the results is obtained. A further check is to compute the amplitude of each mode (an intermediate step in the computation of (9)) to verify that the modes not considered do not store energy. Results are shown in Fig. 7 and compared with those of Marcuvitz.

In order to evaluate the increase in efficiency of a TLM simulation due to this new approach, a discontinuity of arbitrary shape has been analyzed with the classical approach (single-mode Johns matrix placed far away from the discontinuity) and the modal approach (Fig. 9). The results for such a discontinuity are shown in Fig. 8. In the classical approach particular care has been devoted to the spectrum of the chosen

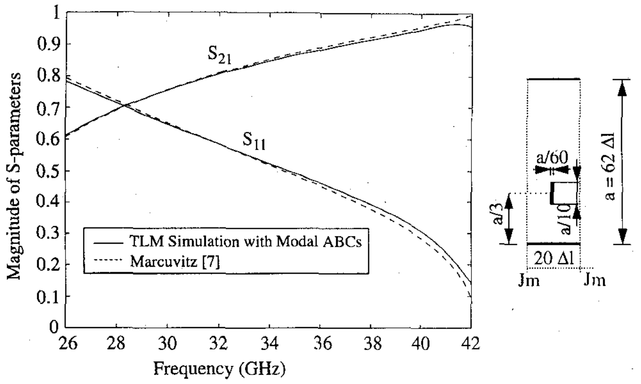

Fig. 7. Magnitude of S-parameters for a transverse post.

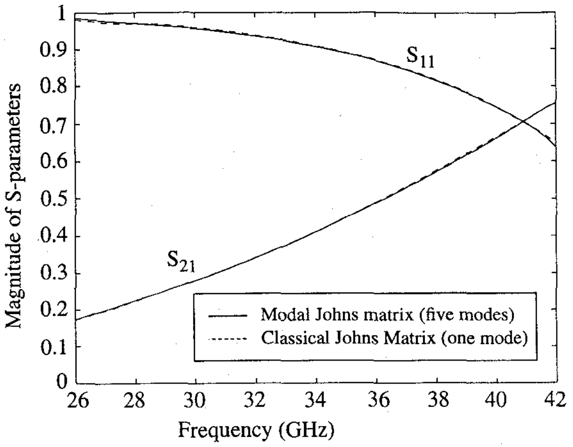

Fig. 8. Magnitude of S-parameters for the 45° inclined iris.

excitation. In fact the bandlimited excitation must not extend beyond the cutoff frequency of the second mode, so that the second mode is not propagating. The distance at which the single mode Johns matrix has been placed has been determined such that the second mode field has decayed to at least 1% of the initial value. Mesh size and CPU time (on an HP9000-755 workstation) for the different cases are summarized in Table I.

In the table a node-to-node convolution (full convolution) as described in [3] has been included for completeness. The savings in memory and CPU time obtained with the new approach are of nearly one order of magnitude if compared to the classical approach, and two orders of magnitude if compared to a full convolution.

It is expected that the experience gained during these simulations will be precious in three-dimensional problems where the saving should be even larger than in the two-dimensional case.

IV. Conclusion

A new wideband absorbing boundary able to handle higher order modes (above or below cutoff) in homogeneous waveguides has been introduced. The results obtained show that the new modal approach is very stable and insensitive to excitation. By using modal absorbing boundaries in the time domain we need to discretize only the region around the discontinuity (where the field is very complex) and treat homogeneous subregions through their modal response.

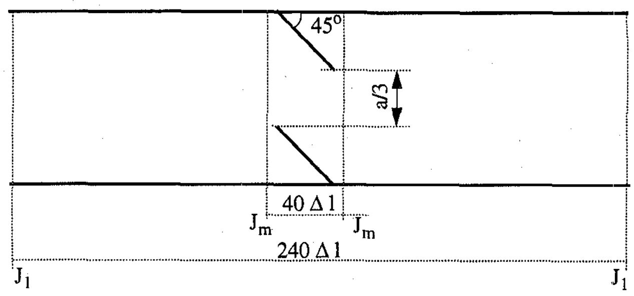

Fig. 9. Inclined iris discontinuity in rectangular waveguide. J_1 indicates the position of the classical Johns matrix boundary (one mode ABC as described in [4]). J_m indicates the position of the modal Johns matrix boundary (five modes).

TABLE I
Comparison of Mesh Size and CPU Time for Single Mode, Multi-Mode, and Node-To-Node ABC's for the Analysis of the 45° Inclined Iris

Type of ABC	Mesh Size in Δl	CPU time
Full Convolution	62 × 30	~ 1 h.
Classical J_1 (one mode)	62 × 240	250 sec.
Multimodal J_m (five modes)	62 × 30	30 sec.

Acknowledgment

The authors wish to thank C. Eswarappa for many discussions and comments on this paper.

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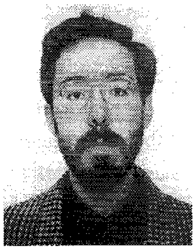

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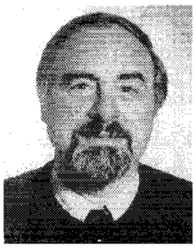

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